This chapter will show you ...

- how to bisect a line and an angle
- how to construct perpendiculars
- how to define a locus
- how to solve locus problems

What you should already know

- How to construct triangles using a protractor and a pair of compasses
- How to use scale drawings

Quick check

Construct these triangles using a ruler, protractor and a pair of compasses.

1

\[ \text{4 cm} \quad \begin{array}{c} 52^\circ \quad 6 \text{ cm} \end{array} \]

2

\[ \text{4 cm} \quad \begin{array}{c} 65^\circ \quad 75^\circ \quad 4 \text{ cm} \end{array} \]

3

\[ \text{5 cm} \quad \text{4 cm} \quad \begin{array}{c} \quad 6 \text{ cm} \end{array} \]
9.1 Bisectors

In this section you will learn how to:
- bisect a line and an angle

Key words
- angle bisector
- bisector
- line bisector
- perpendicular bisector

To bisect means to divide in half. So a bisector divides something into two equal parts.

- A line bisector divides a straight line into two equal lengths.
- An angle bisector is the straight line which divides an angle into two equal angles.

EXAMPLE 1
To construct a line bisector

It is usually more accurate to construct a line bisector than to measure its position (the midpoint of the line).

Bisect the line AB.

- Open your compasses to a radius of about three quarters of the length of the line. Using A and B as centres, draw two intersecting arcs without changing the radius of your compasses.

- Join the two points at which the arcs intersect to meet AB at X. This line is known as the perpendicular bisector of AB.

X is the midpoint of AB.
In this exercise, it is important to leave in all your construction lines.

1. Draw a line 7 cm long. Bisect it using a pair of compasses and a ruler only. Check your accuracy by measuring to see if each half is 3.5 cm.

2. a. Draw any triangle whose sides are between 5 cm and 10 cm.
   b. On each side construct the perpendicular bisector as on the diagram. All your perpendicular bisectors should intersect at the same point.
   c. Use this point as the centre of a circle that touches each vertex of the triangle. Draw this circle. This circle is known as the circumscribed circle of the triangle.

3. Repeat question 2 with a different triangle and check that you get a similar result.

4. a. Draw a quadrilateral whose opposite angles add up to 180°.
   b. On each side construct the perpendicular bisectors. They all should intersect at the same point.
   c. Use this point as the centre of a circle that touches the quadrilateral at each vertex. Draw this circle.

5. a. Draw an angle of 50°.
   b. Construct the angle bisector.
   c. Use a protractor to check how accurate you have been. Each angle should be 25°.
a Draw any triangle whose sides are between 5 cm and 10 cm.

b At each angle construct the angle bisector as on the diagram. All three bisectors should intersect at the same point.

c Use this point as the centre of a circle that touches each side of the triangle once. Draw this circle. This circle is known as the *inscribed circle* of the triangle.

Repeat question 6 with a different triangle and check that you get a similar result.

### 9.2 Other angle constructions

In this section you will learn how to:
- construct perpendiculars from a point
- construct an angle of 60°

**Example 3**

To construct a perpendicular from a point on a line

This construction will produce a perpendicular from a point A on a line.

- Open your compasses to about 2 or 3 cm. With point A as centre, draw two short arcs to intersect the line at each side of the point.
- Now extend the radius of your compasses to about 4 cm. With centres at the two points at which the arcs intersect the line, draw two arcs to intersect at X above the line.
- Join AX.

AX is perpendicular to the line.

**Note:** If you needed to construct a $90^\circ$ angle at the end of a line, you would first have to extend the line.

You could be even more accurate by also drawing two arcs underneath the line, which would give three points in line.
EXAMPLE 4

To construct a perpendicular from a point to a line

This construction will produce a perpendicular from a point A to a line.

1. With point A as centre, draw an arc which intersects the line at two points.
2. With centres at these two points of intersection, draw two arcs to intersect each other both above and below the line.
3. Join the two points at which the arcs intersect. The resulting line passes through point A and is perpendicular to the line.

Examination note: When a question says construct, you must only use compasses – no protractor. When it says draw, you may use whatever you can to produce an accurate diagram. But also note, when constructing you may use your protractor to check your accuracy.

EXAMPLE 5

To construct an angle of 60°

This construction will produce an angle of 60° from a point A on a line.

1. Open your compasses to about 3 cm. With point A as centre, draw an arc from above to intersect the line at point B.
2. With point B as centre, draw a second arc which passes through point A to intersect the first arc at point C.
3. Join AC.

\[ \angle CAB = 60° \]
In this exercise, it is important to leave in all your construction lines.

1. Construct these triangles accurately without using a protractor.

   ![Triangles](image1)

2. a) Without using a protractor, construct a square of side 6 cm.
   b) See how accurate you have been by constructing an angle bisector on any of the right angles and seeing whether this also cuts through the opposite right angle.

3. a) Construct an angle of 90°.
   b) Bisect this angle to construct an angle of 45°.

4. a) Construct these angles. i) 30° ii) 15° iii) 22.5° iv) 75°
   b) Check your accuracy by measuring with a protractor. (The allowable error is ±1°.)

5. With ruler and compasses only, construct these triangles.

   ![Triangles](image2)

6. Construct an isosceles triangle ABC, where AB = AC = 7 cm and ∠CAB = 120°.

7. Construct a trapezium whose parallel sides are 8 cm and 6 cm, and having an angle of 60° at each end of the longer side.

8. a) Construct the triangle ABC, where AB = 7 cm, ∠BAC = 60° and ∠ABC = 45°.
   b) Measure the lengths of AC and BC.

9. a) Construct the triangle PQR, where PQ = 8 cm, ∠RPQ = 30° and ∠PQR = 45°.
   b) Measure the lengths of PR and RQ.

10. Construct a parallelogram which has sides of 6 cm and 8 cm and with an angle of 105°.

11. Draw a straight line and mark a point above the line. Construct the perpendicular which passes through that point to the line.
9.3 Defining a locus

In this section you will learn how to:
● draw a locus for a given rule

Key words
loci
locus

A locus (plural loci) is the movement of a point according to a given rule.

EXAMPLE 6
A point P that moves so that it is always at a distance of 5 cm from a fixed point A will have a locus that is a circle of radius 5 cm.
You can express this mathematically by saying
the locus of the point P is such that AP = 5 cm.

EXAMPLE 7
A point P that moves so that it is always the same distance from two fixed points A and B will have a locus that is the perpendicular bisector of the line joining A and B.
You can express this mathematically by saying
the locus of the point P is such that AP = BP.

EXAMPLE 8
A point that moves so that it is always 5 cm from a line AB will have a locus that is a racetrack shape around the line.
This is difficult to express mathematically.

In your GCSE examination, you will usually get practical situations rather than abstract mathematical ones.

EXAMPLE 9
A point that is always 5 m from a long, straight wall will have a locus that is a line parallel to the wall and 5 m from it.
A is a fixed point. Sketch the locus of the point P in each of these situations.

- **a** $AP = 2\text{ cm}$
- **b** $AP = 4\text{ cm}$
- **c** $AP = 5\text{ cm}$

A and B are two fixed points 5 cm apart. Sketch the locus of the point P for each of these situations.

- **a** $AP = BP$
- **b** $AP = 4\text{ cm}$ and $BP = 4\text{ cm}$
- **c** P is always within 2 cm of the line AB

A horse is tethered in a field on a rope 4 m long. Describe or sketch the area that the horse can graze.

The horse is still tethered by the same rope but there is now a long, straight fence running 2 m from the stake. Sketch the area that the horse can now graze.

ABCD is a square of side 4 cm. In each of the following loci, the point P moves only inside the square. Sketch the locus in each case.

- **a** $AP = BP$
- **b** $AP < BP$
- **c** $AP = CP$
- **d** $CP < 4\text{ cm}$
- **e** $CP > 2\text{ cm}$
- **f** $CP > 5\text{ cm}$

One of the following diagrams is the locus of a point on the rim of a bicycle wheel as it moves along a flat road. Which is it?

Draw the locus of the centre of the wheel for the bicycle in question 6.
Most of the loci problems in your GCSE examination will be of a practical nature, as in the next example.

**EXAMPLE 11**

Imagine that a radio company wants to find a site for a transmitter. The transmitter must be the same distance from Doncaster and Leeds and within 20 miles of Sheffield.

In mathematical terms, this means they are concerned with the perpendicular bisector between Leeds and Doncaster and the area within a circle of radius 20 miles from Sheffield.

The map, drawn to a scale of 1 cm = 10 miles, illustrates the situation and shows that the transmitter can be built anywhere along the thick part of the blue line.

**EXERCISE 9D**

For questions 1 to 7, you should start by sketching the picture given in each question on a $6 \times 6$ grid, each square of which is 1 cm by 1 cm. The scale for each question is given.

1. A goat is tethered by a rope, 7 m long, in a corner of a field with a fence at each side. What is the locus of the area that the goat can graze? Use a scale of 1 cm = 2 m.

2. In a field a horse is tethered to a stake by a rope 6 m long. What is the locus of the area that the horse can graze? Use a scale of 1 cm = 2 m.
A cow is tethered to a rail at the top of a fence 6 m long. The rope is 3 m long. Sketch the area that the cow can graze. Use a scale of 1 cm ≡ 2 m.

A horse is tethered to a stake near a corner of a fenced field, at a point 4 m from each fence. The rope is 6 m long. Sketch the area that the horse can graze. Use a scale of 1 cm ≡ 2 m.

A horse is tethered to a corner of a shed, 2 m by 1 m. The rope is 2 m long. Sketch the area that the horse can graze. Use a scale of 1 cm ≡ 1 m.

A goat is tethered by a 4 m rope to a stake at one corner of a pen, 4 m by 3 m. Sketch the area of the pen on which the goat cannot graze. Use a scale of 1 cm ≡ 1 m.

A puppy is tethered to a stake by a rope, 1.5 m long, on a flat lawn on which are two raised brick flower beds. The stake is situated at one corner of a bed, as shown. Sketch the area that the puppy is free to roam in. Use a scale of 1 cm ≡ 1 m.

For questions 8 to 15, you should use a copy of the map opposite. For each question, trace the map and mark on those points that are relevant to that question.

A radio station broadcasts from London on a frequency of 1000 kHz with a range of 300 km. Another radio station broadcasts from Glasgow on the same frequency with a range of 200 km.

a Sketch the area to which each station can broadcast.
b Will they interfere with each other?
c If the Glasgow station increases its range to 400 km, will they then interfere with each other?

The radar at Leeds airport has a range of 200 km. The radar at Exeter airport has a range of 200 km.

a Will a plane flying over Birmingham be detected by the Leeds radar?
b Sketch the area where a plane can be picked up by both radars at the same time.
A radio transmitter is to be built according to these rules.

i. It has to be the same distance from York and Birmingham.

ii. It must be within 350 km of Glasgow.

iii. It must be within 250 km of London.

a. Sketch the line that is the same distance from York and Birmingham.

b. Sketch the area that is within 350 km of Glasgow and 250 km of London.

c. Show clearly the possible places at which the transmitter could be built.

A radio transmitter centred at Birmingham is designed to give good reception in an area greater than 150 km and less than 250 km from the transmitter. Sketch the area of good reception.

Three radio stations pick up a distress call from a boat in the Irish Sea. The station at Glasgow can tell from the strength of the signal that the boat is within 300 km of the station. The station at York can tell that the boat is between 200 km and 300 km from York. The station at London can tell that it is less than 400 km from London. Sketch the area where the boat could be.

Sketch the area that is between 200 km and 300 km from Newcastle upon Tyne, and between 150 km and 250 km from Bristol.

An oil rig is situated in the North Sea in such a position that it is the same distance from Newcastle upon Tyne and Manchester. It is also the same distance from Sheffield and Norwich. Draw the line that shows all the points that are the same distance from Newcastle upon Tyne and Manchester. Repeat for the points that are the same distance from Sheffield and Norwich and find out where the oil rig is located.

Whilst looking at a map, Fred notices that his house is the same distance from Glasgow, Norwich and Exeter. Where is it?

Wathsea Harbour is as shown in the diagram.

A boat sets off from point A and steers so that it keeps the same distance from the sea wall and the West Pier. Another boat sets off from B and steers so that it keeps the same distance from the East Pier and the sea wall. Copy the diagram below, and on your diagram show accurately the path of each boat.

The curve $x^2 + y^2 = 25$ is a circle of radius 5 centred on the origin.

a. Show that the points (3, 4) and (-4, 3) lie on the curve.

b. Sketch the loci of the curve $x^2 + y^2 = 16$ showing clearly the values where it crosses the axes.
Make an accurate drawing of this triangle.

Construct an accurate drawing of this triangle.

Construct an angle of 60°.

Copy the line AB and then construct the perpendicular bisector of the points A and B.

The map shows a small island with two towns A and B. Town B is north west of town A. The map is drawn to a scale of 1 square to 10 km.

a What bearing is the direction north west?

b A mobile phone mast is to be built. It has to be within 40 km of both towns. Copy the map and shade the area in which the mast could be built.

The diagram represents a triangular garden ABC. The scale of the diagram is 1 cm represents 1 m. A tree is to be planted in the garden so that it is nearer to AB than to AC, within 5 m of point A. Copy the diagram and shade the region where the tree may be planted.
Use a ruler and compasses to construct the perpendicular from P to the line segment XY. You must show all construction lines.

This is a map of part of Northern England.

A radio station in Manchester transmits programmes. Its programmes can be received anywhere within a distance of 30 km.

On a copy of the diagram, shade the region in which the programmes can be received.

Edexcel, Question 2, Paper 6 Higher, June 2004

Use a ruler and compasses to construct the bisector of angle ABC. You must show all construction lines.

The diagram shows three points A, B and C on a centimetre grid.

a) On a copy of the grid, draw the locus of points which are equidistant from A and B.

b) On a copy of the grid, draw the locus of points that are 3 cm from C.

c) On a copy of the grid, shade the region in which points are nearer to A than B and also less than 3 cm from C.

Edexcel, Question 1, Paper 13A Higher, January 2004
WORKED EXAM QUESTION

The map shows two trees, A and B, in a park. At the edge of the park there is a straight path. A new tree, C, is to be planted in the park. The tree must be:
- more than 60 m from the path,
- closer to A than B,
- more than 100 m from A.

Using a ruler and compasses only, shade the region where the tree could be planted. You must show all construction lines.

Solution

1. Draw a parallel line 3 cm from the path.
2. Draw the perpendicular bisector of AB.
3. Draw a circle of radius 5 cm at A.

The region required is shaded on the diagram.
Bill the builder builds a street of 100 bungalows, 50 on each side of the street. He builds them in blocks of 5. The bungalows at the end of the blocks are called end-terraced, and the other bungalows are called mid-terraced.

A tree is to be planted in the back garden of each mid-terraced house. The tree must be at least 2 m from the back of the house, at least 1 m from the back fence of the garden, and at least 3.5 m from each of the bottom corners of the garden. It must also be at least 1.5 m from the garden shed.

Draw an accurate scale drawing, using a scale of 1 cm = 1 m, of a mid-terraced house and garden. Shade the region in which Bill can plant the tree.
Bill needs to work out the number and cost, as well as the weight of the roof slates needed for the whole street.

Each roof slate costs 24p, 17.4 slates cover one square metre of roof, and 1000 slates weigh 2400 kg.

Using the dimensions on the plan and the height given on the first block shown on the left, help Bill to fill in the table. Give the areas to the nearest square metre.

<table>
<thead>
<tr>
<th>Roof area for one block of 5 bungalows</th>
<th>m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof area of whole street</td>
<td>m²</td>
</tr>
<tr>
<td>Number of slates needed</td>
<td>slates</td>
</tr>
<tr>
<td>Total cost of slates</td>
<td>£</td>
</tr>
<tr>
<td>Total weight of slates</td>
<td>kg</td>
</tr>
</tbody>
</table>
GRADE YOURSELF

- Able to construct line and angle bisectors
- Able to draw and describe the locus of a point from a given rule
- Able to solve problems using loci
- Able to construct a perpendicular from a point on a line
- Able to construct a perpendicular from a point to a line
- Able to construct an angle of 60°

What you should know now

- How to construct line and angle bisectors
- How to construct perpendiculars
- How to construct angles without a protractor
- Understand what is meant by a locus
- How to solve problems using loci